QP CODE: 26018591

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Reg No : .....

Name : .....

MSc DEGREE (CSS) SPECIAL REAPPEARANCE EXAMINATION, MARCH 2026

Third Semester

M.Sc. MATHEMATICS

CORE COURSE - ME010301 - ADVANCED COMPLEX ANALYSIS

2024 Admission Only

17E0D677

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define a harmonic function and give an example.
2. State Reflection principle.
3. Find the Laurent series expansion of $f(z) = \frac{1}{z(z-1)^2}$ about the point $z = 1$.
4. Find the poles of $\frac{\pi^2}{sin^2 \pi z}$ and the corresponding singular parts.
5. Find the value of $\Gamma(z)$ at $z = 1$ and $z = \frac{1}{2}$.
6. What do you mean by the trivial zeros of the Riemann zeta function?
7. Prove that a family of functions is normal then its closure with respect to distance function is compact.
8. Let $\mathcal{F}$ be a normal family of functions in Ω with values in a metric space S. Prove that $\mathcal{F}$ is equicontinuous on every compact subset $E \subseteq \Omega$.
9. What is meant by a Canonical basis?
10. State and prove the Legendre's relation for the ζ - function.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. State and prove Poisson's formula.
12. *Prove that if V is subharmonic in a region Ω then the function V' defined as P_v in Δ and V outside of Δ is also subharmonic, where Δ is an open disk whose closure is contained in Ω :*

13. State and prove Hurwitz theorem.

14. Prove that a necessary and sufficient condition for the absolute convergence of the product $\Pi_1^{\infty}(1+a_n)$ is the convergence of the series $\Sigma_1^{\infty}|a_n|$.

15. Derive the functional equation of the Riemann Zeta function.

16. *Find the sum of residues of the function* $f(z) = \frac{(-z)^{s-1}}{e^z-1}$.

17. Let Ω be a simply connected region other than the complex plane. Prove that the Riemann mapping from Ω to the unit disk is one to one.

18. Let f be a topological mapping of a region Ω onto a region Ω'. If $\{z_n\}$ or $z(t)$ tends to the boundary of Ω, then prove that $\{f(z_n)\}$ or $f(z(t))$ tends to the boundary of Ω'.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (a) When do you say that a real valued function $u(z)$ satisfies mean value property in a region Ω ?
(b) Prove that a continuous function $u(z)$ satisfying the mean value property is harmonic.
(c) State and prove the Harnack's inequality.

20. Let $f(z)$ be analytic in the closed disc $|z| \leq \rho$. Assume $f(0) \neq 0, f(z) \neq 0$ on $|z| = \rho$.. If $a_1, a_2, \ldots a_n$ are the zeros of $f(z)$ in $|z| \leq \rho$,multiple zeros being repeated, then prove that $log|f(0)| = -\Sigma_{i=1}^{n} log\frac{\rho}{|a_i|} + \frac{1}{2\pi}\int_0^{2\pi} log|f(\rho e^{i\theta})|d\theta$

21. (i) Prove that the Riemann Zeta function is analytic in the half plane $Re\ s > 1$.
(ii) Derive the product representation of the Zeta function.
(iii) Derive the integral representation of the Zeta function.

22. **(a)Prove that** $\wp(z+u) + \wp(z) + \wp(u) = \frac{1}{4}[\frac{\wp'(z)-\wp'(u)}{\wp(z)-\wp(u)}]^2$.

(b) Prove that $\wp'(z) = -\frac{\sigma(2z)}{\sigma(z)^4}$.

(c)Prove that $\begin{vmatrix} \wp(z) & \wp'(z) & 1 \\ \wp(u) & \wp'(u) & 1 \\ \wp(z+u) & -\wp'(z+u) & 1 \end{vmatrix} = 0.$

(2×5=10 weightage)